Developing Earth models with full waveform inversion

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Exploration in more geologically complex areas requires new methodologies. In its quest to answer these new challenges, the oil and gas industry has moved from ray-based imaging to finite-difference, wave-equation migration to achieve better subsurface descriptions of target zone and reservoirs. Notable in this progression is the movement from ray-traced Kirchhoff algorithms through one-way wave-equation methods to use the acoustic two-way wave equation.

Although migration has advanced quickly with increased computer power, constructing the Earth model is still largely ray-based, using gathers from advanced migration algorithms. Recently, there has been more emphasis on using the two-way wave equation for migration and for velocity model building. One advanced tool for velocity determination is full waveform inversion.

Prestack full waveform inversion is highly challenging due to the nonlinearity and nonuniqueness of the solution. When combined with compute-intensive forward modeling and residual wavefield back propagation, the method is resource and time-consuming—especially for 3D projects. The implementations in the frequency or time domain have advantages, so the question is which is viable to execute in 3D on current hardware. Since the two-way wave equation is widely used for migration algorithms, one can assume this would point us to the time-domain solution. The time-domain method is expensive due to the forward and time-reversed wavefield propagation cost using a given time interval to avoid dispersion and aliasing. However, cluster computers are efficient for most processing steps, including reverse time migration. This lets us perform time-domain waveform inversion of large 3D data sets on current hardware.

Full waveform inversion in the time domain

The inversion has three key steps:

1) Calculating the differences between the acquired data and the current model through forward modeling to check the accuracy of the current model.
2) Cross-correlating the back-propagated residual wavefield with the corresponding forward source-propagated wavefield at each time step and summing over all time steps to produce the gradient volume.
3) Making the amplitude of the gradient at each spatial point proportional to the velocity change.

Seismic waveform inversion is based on the minimization of a cost function which measures the difference between the calculated and acquired data. In order to minimize the cost function, several forward modeling and residual back propagations are required to gradually update the velocity field. Our optimized time-domain approach in the plane-wave or shot domain makes waveform inversion feasible for large 3D surveys. By performing waveform inversion in a multiscale manner, lower frequencies can provide several scalable options, such as changing the shots in conjunction with incrementing the bin sizes in all directions. These features are essential to run 3D inversions relatively quickly. In addition, the inversion can be carried out in a layer-stripping style to accommodate the traditional model-building flows and control/speed up the time needed to derive a model. The velocity update is fine-tuned by a line search to determine the optimum velocity update using the derived gradient at a given iteration. The line search involves evaluation of several misfit functions ($E=\frac{1}{2}\sum_s\sum_r\int dt[P_{cal}(\mathbf{x}_r,t)-P_{obs}(\mathbf{x}_r,t)]^2$) in which we need two additional values of α such that α_1 is less than α_2 and $E(1)$ is less than both $E(0)$ and $E(2)$. $E(0)$ has already been evaluated from the gradient calculation (Figure 1). Each additional misfit function evaluation needs a forward modeling run to calculate the residuals required for the function. When we have three valid values, we use the minimum of the parabolic function describing these points. The velocity is updated with the formula $v_{n+1}=v_n+\alpha\gamma_n$ where α is a step length.

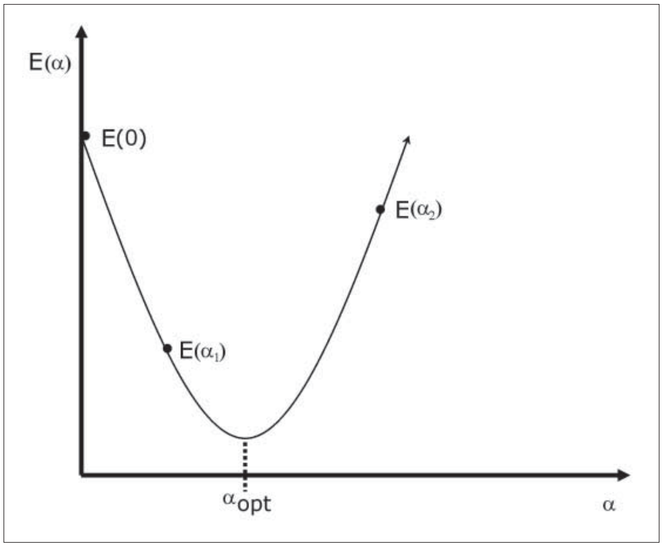

Figure 1. *Step length line search using three points.*

Inversion proceeds in a multiscale approach (Bunks, 1995) from lower to higher frequencies in an attempt to improve the chances that the global minimum is reached and not a local one. The multiscale temporal decomposition applies a windowed finite-impulse response (FIR) low-pass sinc filter (Oppenheim and Schafer, 1975) to the modeled and observed data. In our multiscale approach, the inversion results from the lower frequencies are passed on to the next higher range of frequencies while the lowest frequency used in each scale step is held constant from scale step to scale step. In this case, our time-domain method does not take advantage of wavenumber redundancy that frequency-domain methods use to limit frequencies. However, using redundant coverage may make our method more robust. Redundant coverage may be

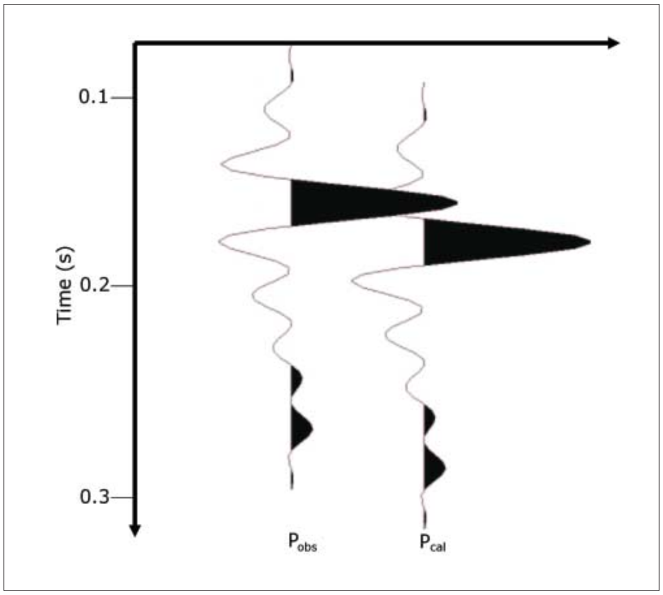

Figure 2. Observed and calculated data within half a period of the dominant wavelength.

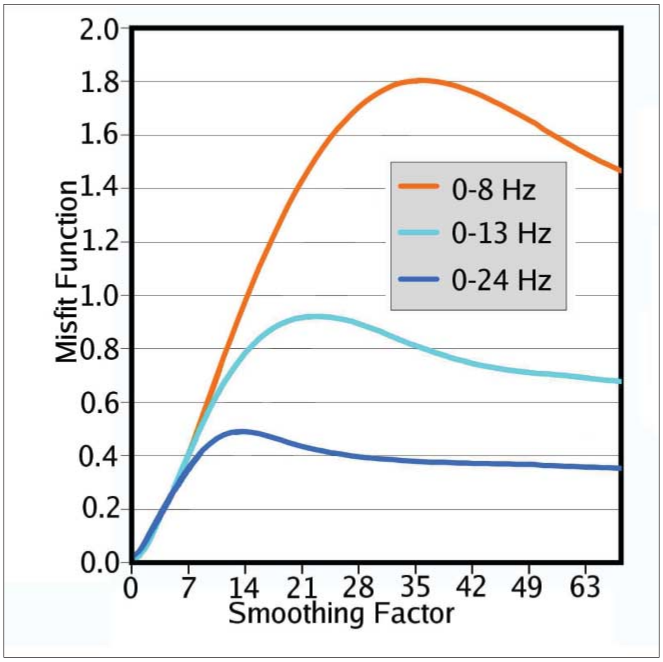

Figure 3. Smoothing factor versus misfit function over different frequency ranges using a known velocity from the Pluto 1.5 data set.

needed, for instance, to improve the signal-to-noise ratio.

Nonlinearity of the waveform inversion problem

All waveform inversions using a gradient method should provide a long-wavelength velocity model that is within half a period of the dominant wavelength to assure convergence (Figure 2) to a global minimum. This issue of nonlinearity in the *x-t* domain between the observed data and the model data might be seen as cycle skipping.

When convergence to local minima occurs, one has to use lower frequencies or start with a better higher-frequency velocity model. To prevent local minima, we have to determine how accurate the initial velocity model and the starting frequency range must be by analyzing the misfit function versus the smoothness using a known true-velocity model. As shown in Figure 3, we progressively increased the smoothing on a known velocity model and measured the misfit for different frequency ranges. When the slope of a curve proceeds downward to the right, the solution will converge to local minima.

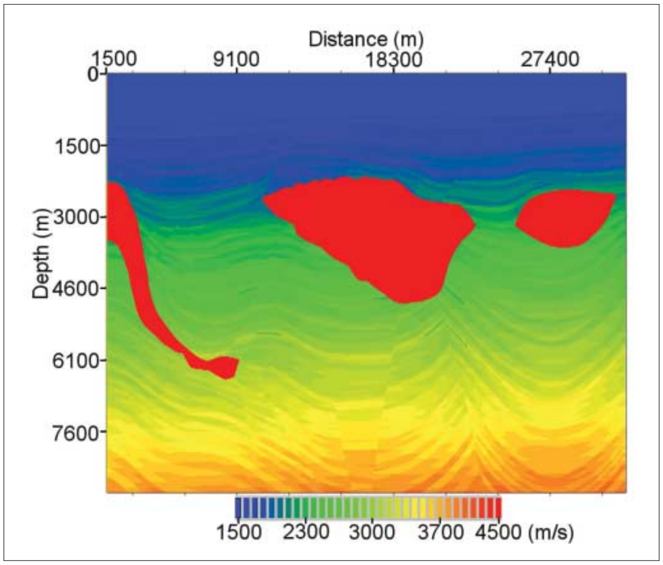

Figure 4. Pluto true velocity.

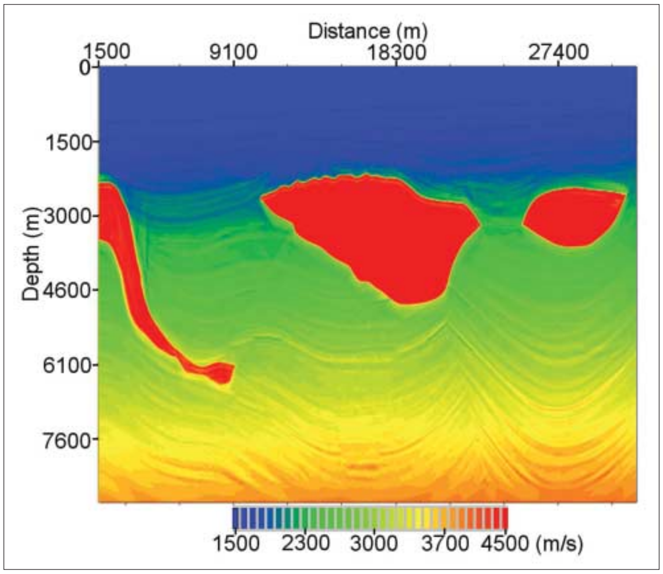

Figure 5. Pluto after waveform inversion.

Data preconditioning

Important preprocessing steps in the inversion process include:

- Wavelet estimation from the data and zero-phasing in conjunction with bubble removal in the marine data case. The source wavelet for the inversion should, as closely as possible, approximate the one in the seismic data. This is a key step prior to the expensive inversion process.

- Since the forward modeling is free of random and coherent noise, it is advised to remove these phenomena from the real data before the inversion. If enough noise is present, the waveform inversion will attempt to incorrectly change the inverted attribute so that the modeled data match the observed data in a least squares sense.
- Multiple attenuation is the most controversial issue since the modeling can generate them and could further condition the inversion flow. However, practice still shows waveform inversion and migration benefits from multiple removal, whether it is free surface or internal.
- Amplitude irregularities should be edited prior to the inversion. This may occur when the strength of the gun changes or some receivers get weaker than others during the acquisition. Surface-consistent gain application is recommended to remove these effects.

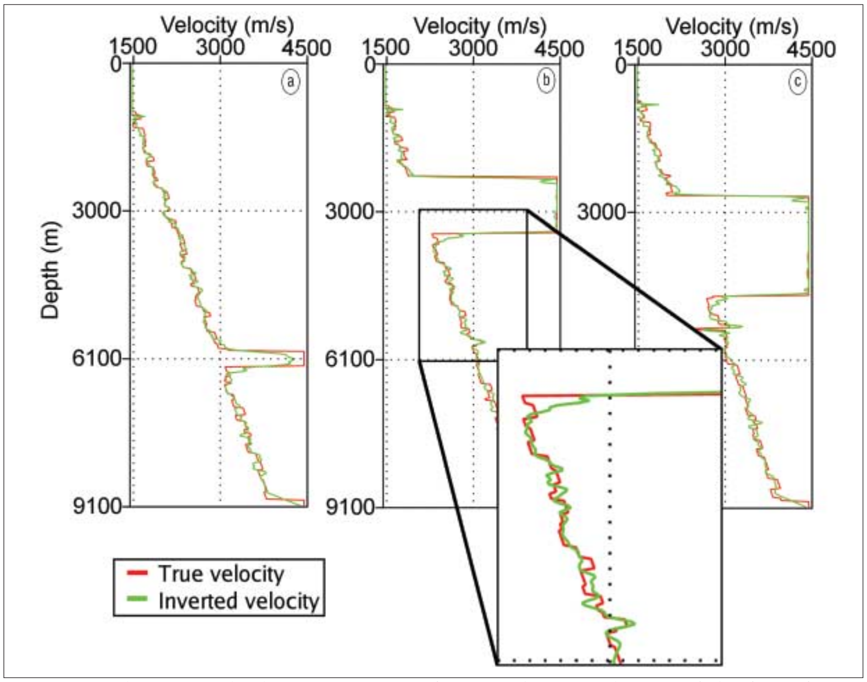

Figure 6. Pluto vertical velocity comparisons between the true velocity and the derived velocity at (a) distance 6530 m, (b) distance 12,820 m, and (c) distance 20,360 m.

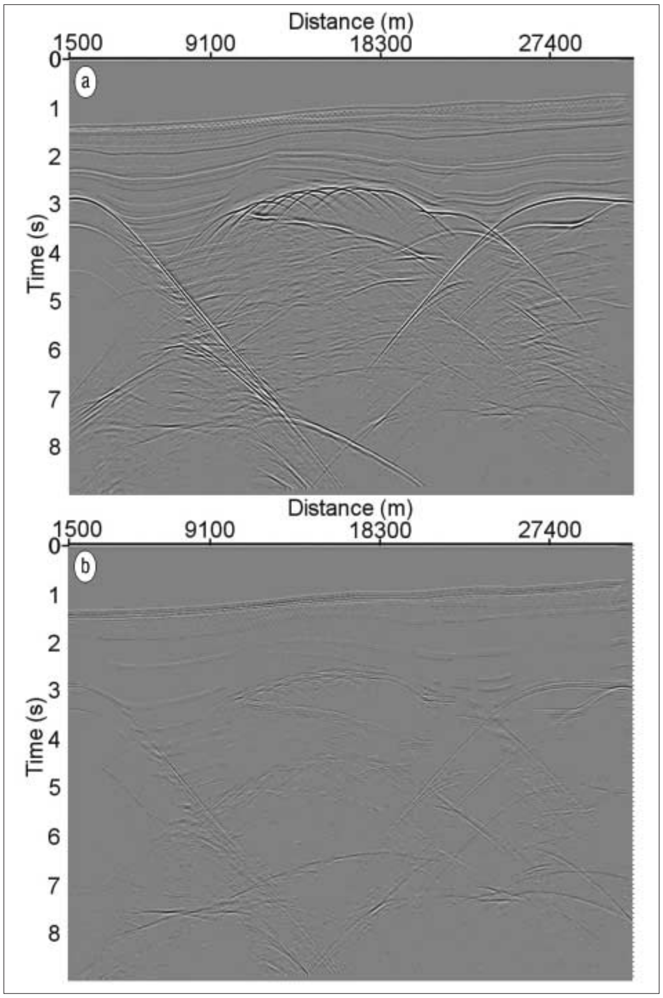

Figure 7. Pluto data residuals at a near offset for the (a) starting velocity and (b) the final inverted velocity. The same display gain is applied to both sections.

Synthetic data example

The SMAART Pluto 1.5 model was used to validate the full waveform inversion on synthetic data. The original model had the free-surface multiples turned on during the forward modeling; therefore, the data were reshot to exclude them to avoid SRME. The data were regenerated using the pressure velocity field and the density with a 40-Hz maximum frequency Ricker wavelet. The shots were acquired on the original grid (23 m for the receiver and shot intervals). The inversion started with a smooth version of the original pressure velocity field and three frequency bands (0–13, 0–18, and 0–24 Hz) starting with the lower band and marching towards the higher ones and minimizing the misfit function in each band throughout numerous iterations. The true velocity is shown in Figure 4, and the inverted one, after tens of iterations in each frequency band is shown in Figure 5. After 57 iterations across the different frequency bands, the final inverted velocity field was determined. The vertical comparisons at selected CDP locations are seen in Figure 6. The match between the true and the inverted velocity field is fairly good whether the velocity section or the vertical comparison is used. The data space residuals are proportional to the misfit energy, so analyzing the data-space residuals could be a good indicator of convergence of the accuracy of the velocity field at a given iteration. The data-space residual decreased substantially over the iterations. The remaining energy can be explained by the small discrepancy between the true and final inverted velocity model, especially at the salt boundary. The inverted model is not as crisp as the true model which reduces the calculated reflection coefficients. This difference is noticeable on Figure 7.

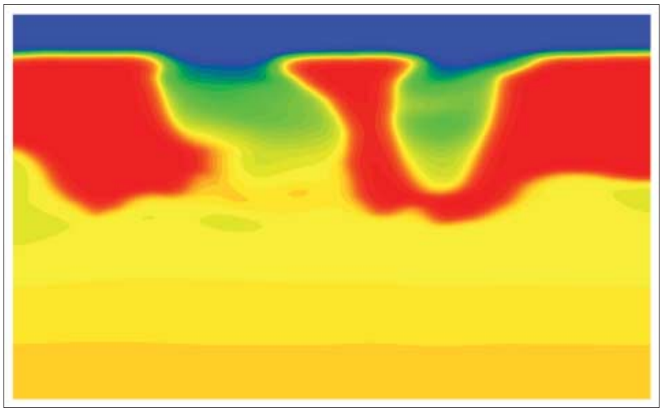

Figure 8. 3D wide-azimuth starting velocity.

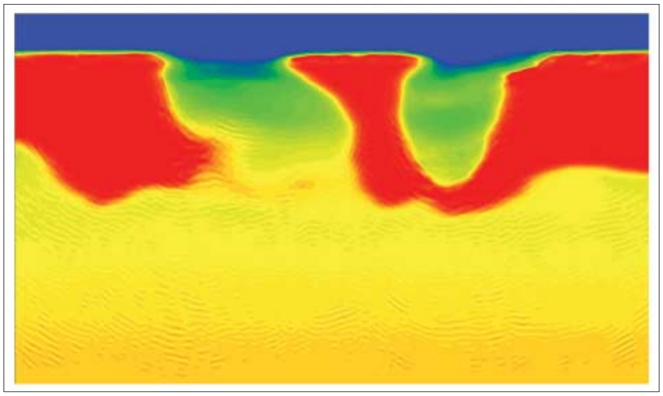

Figure 9. 3D wide-azimuth inverted velocity.

Field data example

The 3D marine data set is typical for the Gulf of Mexico. Free-surface multiples have been removed using an SRME algorithm. The source wavelet was derived using the near offsets at the water bottom and a shaping filter to transform the data to a maximum frequency 40-Hz Ricker wavelet. The inversion was executed on about 220 square miles of wide-azimuth data. Prior to the inversion, the data were transformed to "super-shots" to significantly reduce the number of shots across the survey. Due to the significant crossline offset and the reduced number of shots, the waveform inversion was performed in the shot-gather domain. The starting velocity model has some initial knowledge of the salt-body from the traditional model-building flow. Because of uncertainties in the interpretation, however, significant smoothing was applied to this velocity representation (Figure 8). The first 10 iterations were carried out in the frequency band of 0–7 Hz in order to capture the low-frequency component of the Earth model (Figure 9). The big guns and the single-receiver arrays used during acquisition allowed using very valuable frequencies of about 3 Hz in the recorded data for the inversion. The other advantage of the WAZ data acquisition is better illumination, another essential need for successful waveform inversion. During the iterations, a line search was used to aid faster convergence. This method optimizes the magnitude of the update that is proportional to the gradient.

Summary

In mature exploration areas such as the Gulf of Mexico, fairly good velocity fields derived through typical PSDM workflow can help create a relatively good starting velocity volume which will lessen the effect of the possible lack of low frequencies in the acquired data. Promising new data collection methods can extend the low-frequency part of the spectrum by a few Hz which could help out the inversion process. If the number of iterations is within a reasonable range, 3D inversions are feasible on the latest hardware configurations. Even if the human component (interpretation and/or image gather analysis) of the PSDM processing sequence could be eased and the subjective human decision can be augmented by full waveform inversion, then it can play a leading role in building Earth models in a semi-automated way. With the emerging hardware platforms, anisotropic parameters can be added which can aid in building more accurate velocity representations.

Suggested reading. "Multiscale seismic waveform inversion" by Bunks et al. (Geophysics, 1995). "Efficient waveform inversion and imaging: a strategy for selecting temporal frequencies" by Sirgue and Pratt (Geophysics, 2004). "3D finite difference frequency-domain modeling of visco-acoustic wave propagation using a massively parallel direct solver: A feasibility study" by Operto et al. (Geophysics, 2007).

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